Essentials of Standard ML Modules

Mads Tofte

Department of Computer Science
University of Copenhagen

Abstract. The following notes give an overview of Standard ML Modules system.

Part 1 gives an introduction to ML Modules aimed at the reader who is familiar with a functional programming language but has little or no experience with ML programming.

Part 2 is a half-day practical intended to give the reader an opportunity to modify a small, but non-trivial piece of software using functors, signatures and structures.

PART 1

1 Introduction

It is now more than ten years ago that David MacQueen made his proposal for ML Modules [Mac84]. At the time, there was very little experience with large scale programming in ML. At the time the Modules were formally defined (1987-1989), there was still a certain amount of guesswork involved, still because of the limited practical experience. Today, hundreds of thousands of lines of ML later, ML programmers have a much clearer picture of what the most important aspects of the Standard ML modules are. In the experience of the author, there are certain features of the ML Modules system that are exploited again and again, while others play a strictly secondary role. Moreover, these essential features are actually surprisingly few in number and not hard to grasp. Finally, their scope is not limited to ML; for example, they are completely independent of the fact that ML is a strict (rather than an lazy) language. The purpose of these notes is to focus on these few essentials of ML Modules.

1 By and large, these notes are consistent with The Definition of Standard ML [MTH90], as regards syntax, semantics and terminology. As it happens, the Definition is currently being revised, primarily in order to simplify the modules system. In these notes we concentrate on those aspects of the ML modules that will still be present in the revised language. For brevity, we refer to the old and the revised languages as SML 90 and SML 96, respectively, when the distinction matters.

The exercises in the first part can be solved without the use of a computer; the tutorial assumes that an SML 90 implementation with the Edinburgh Library preloaded is available. The ML code for Part 2 is available from the author’s World-Wide Web home page.
2 Packaging Code using Structures

In programming languages the term *module* means a packaged program unit which can be combined with other modules to form a (possibly large) software system. The adjective *modular* is often used as a positive term, implying tidy design and good software hygiene.

Many functional languages already have a concept of type which is strong enough to allow the orderly organisation of *values*. For example, it is much easier to program with binary trees, if one uses recursive datatypes than if one uses pointers to represent trees. Also, function composition is a way of organising computations (each function is regarded as a computation); type checking can catch meaningless combinations of computations at compile time. Moreover, since the composition of two functions is again a function, which is itself a value, functional languages make it possible to "compute with computations" in an orderly manner. So why add language constructs for modularity?

The reason is that in a typed language one wants an orderly organisation not just of values but also of types. There is a useful distinction between a value and its type; the type is usually much simpler and reveals less detail than the value. Similarly, there is a useful distinction between a particular type (i.e., a choice of data type) and the specification that a type exist and have a certain arity, say. Both forms of information hiding are important in typed languages. In ML, values cannot contain types, so one cannot simply build a record containing a datatype together with some operations on that type. The separation of values and types makes static type checking possible. However, the price for this separation is that one needs separate language constructs for packaging types and values that belong together.

ML allows the programmer to package a collection of types and values into a single unit, called a *structure*. The corresponding notion in ADA is *package*.

Here is a structure, called *IntFn*, which implements finite maps on integers:

```markdown
structure IntFn =
struct
  exception Apply
  type 'a intmap = int \to 'a
  fun c i = raise Apply
  fun app f x = f x
  fun extend (a, b) f i =
    if i=a then b else f i
end;
```

Having declared *IntFn*, we can refer to the types and values it contains using *qualified identifiers*. A qualified identifier starts with a structure name, then comes a period and at the end is a normal identifier.
val a: bool IntFn.intmap = IntFn.e
val b = IntFn.extend (3, true) a
val c = IntFn.app b 3
val d = IntFn.app b 4;

(The type constraint “: bool IntFn.intmap” isn’t really needed, but it illustrates that one can refer to types as well as values.)

Exercise 1. What are the types of b, c, and d?

Below is a signature which specifies the types and values of IntFn without revealing what they are:

signature INTMAP =
sig
  exception Apply
  type 'a intmap
  val e: 'a intmap
  val app: 'a intmap -> int -> 'a
  val extend: int*'a ->
    'a intmap -> 'a intmap
end;

To sum up, a collection of values and types can be packaged into a structure. In the above example, we had just one type in the structure; it is common to introduce several types in a single structure. A signature is a “structure type”, i.e., it classifies structures in analogy with the fact that types classify values.

3 Using Signatures as Interfaces

In typed programming languages, a type checker can ensure that no value is used in a way which conflicts with its type. The same idea is clearly useful at the level of modules. For example, it should be a "type error" to place a structure \( M \) in some context where one actually needs a module which implements more operations than \( M \) does.

When one declares a structure in ML, one can get the compiler to check whether the structure matches a given signature. For example, if we assume that we have first declared signature INTMAP as above, but not yet IntFn, we can declare IntFn as follows:
structure IntFn: INTMAP =
struct
  exception Apply
  type 'a intmap = int -> 'a
  fun ε i = raise Apply
  fun app f x = f x
  fun extend (a, b) f i =
    if i=a then b else f i
end;

The only change is in the first line; the “: INTMAP” is an example of a signature constraint; it makes the compiler check whether the declared structure really matches the signature. Roughly speaking, a structure matches a signature if it has all the types and values specified in the signature. In addition, the types in the structure must have the arities specified in the signature and the values in the structure must have the types specified in the signature. (The structure is allowed to have more values and types than the ones specified by the signature.)

If the structure does not match the signature, an error message is printed. Otherwise, the result of the constrained declaration is that the structure identifier (here IntFn) is bound to a structure which has precisely the values and types specified by the signature (here INTMAP). In other words, after the declaration, one can only refer to those components of the structure that are specified in the signature.

However, a signature constraint does not hide the identity of the types that appear both in the structure and in the signature. Hence, after the above declaration, one can exploit the fact that IntFn really is a function type, so one is allowed to write for example:

val x = IntFn.x 5;

(This will raise an exception, when evaluated, but the declaration is well-typed.) Contrast this with the politically correct:

val y = IntFn.app IntFn.x 5;

which is well-typed even if we only assume the type information which is given in the signature.

The form of matching just described is called transparent matching, since the true identity of types shines through the signature constraint. SML '96 also provides opaque matching, which results in a structure which has precisely the type information and components which are specified in the signature. It uses the keyword :> (read: coerced to) instead of ;, so one can write for example:
structure IntFn := INTMAP =
struct
  exception Apply
  type 'a intmap = int -> 'a
  fun c i = raise Apply
  fun app f x = f x
  fun extend (a,b) f i =
    if i=a then b else f i
end;

after which the declaration of x above would be illegal, whereas the declaration of y would still be legal.

4 An Analogy with Mathematics

The distinction we made in Section 2 (namely between, on the one hand, actual types and values and, on the other hand, the specification of types and values) is not in any way new. Indeed, mathematicians have been doing this sort of thing for centuries. A mathematician introduces the concept of a group roughly like this:

Definition 2. A group \((G, \cdot)\) is a set \(G\) equipped with a composition \(\cdot : G \times G \to G\) which is associative, has a neutral element and satisfies that every element of \(G\) has an inverse.

Shortly after, one might find the following example:

The integers \((\mathbb{Z}, +)\) is a group.

The point is that the definition of groups is independent of which set and which composition is chosen. To specify groups in SML, one declares:

signature GROUP =
sig
  type G
  val e : G
  val bullet : G \* G \to G
  val inv : G \to G
end;

Admittedly, this specification would probably not satisfy a mathematician, since it does not specify the required properties of \(e\), \(bullet\) and \(inv\). However, the advantage of providing only relatively simple forms of specifications is that it is decidable whether a given structure matches a given signature — this is highly
desirable when working with many modules and specifications. The group of integers is now declared as follows, where \( \sim \) means unary minus:

\[
\text{structure } Z : \text{GROUP} = \\
\text{struct} \\
\text{type } G = \text{int} \\
\text{val } e = 0 \\
\text{fun bullet}(n : \text{int}, m) = n + m \\
\text{fun inv}(n : \text{int}) = \sim n \\
\text{end;}
\]

5 Parameterised Modules

The reason group theory is group theory is that it applies to all groups. The mathematicians do not re-invent group theory each time a new group comes along. Using Computer Science jargon, the definition of groups is the interface to group theory. If we want to write code which works for all groups, it suffices to see how mathematicians refer to groups without considering a particular one. They simply say: “Let \((G, \cdot)\) be a group”. This is a very compact way of saying several things at once. First, the statement fixes attention on a hypothetical group and gives it a name. Second, it says that, until further notice, all we may assume about \((G, \cdot)\) is that it is a group. It is an elementary logical mistake to use the members of \(G\) as integers, say, unless the set \(G\) has explicitly been constrained to be the integers.

The way to write an ML module which works for any group is to use a functor, e.g.,

\[
\text{functor } Sg(Gr : \text{GROUP}) : \text{GROUP} = \\
\text{struct} \\
\text{type } G = Gr.G * Gr.G \\
\text{val } e = (Gr.e, Gr.e) \\
\text{fun bullet}((a1, b1), (a2, b2)) = \\
(Gr.bullet(a1, a2), \\
Gr.bullet(b1, b2)) \\
\text{fun inv}(a, b) = (Gr.inv a, Gr.inv b) \\
\text{end;}
\]

Here \(Sg\) is the name of the functor, \(Gr\) is the formal parameter, the first occurrence of \(\text{GROUP}\) is the parameter signature, the rightmost occurrence of \(\text{GROUP}\) is the result signature and the structure expression \text{struct...end} is the body of the functor. Inside the body of the functor, all we may assume about structure \(Gr\) is that it matches the parameter signature. The scope of the specification
of \( \text{Gr} \) is the result signature and the functor body. So in general, a functor declaration

\[
\text{functor } f (X : \Sigma) : \Sigma' = \text{body}
\]

is the ML programmer’s way of saying: “let \( X \) be a structure which matches \( \Sigma \)”.

If we want to write a module which works only for groups over the integers we have to constrain the type \( \text{Gr}.G \) to \text{int} and this has to be done “up front”, when we introduce \( \text{Gr} \) as a formal parameter (i.e., the body of the functor is not allowed to impose type equalities which are not specified in the parameter signatures). In SML 90 one uses a type sharing constraint in the signature:

```
functor Try(Gr:
  sig
    type G
    sharing type G = int
    val e : G
    val bullet : G*G->G
    val inv : G->G
  end) =
struct
  val x = Gr.inv(Gr.bullet(?, 9))
end;
```

In SML 96 one can express the same thing more briefly using a where type qualifier on the signature \text{GROUP}:

```
functor Try(Gr: GROUP
  where type G = int) =
struct
  val x = Gr.inv(Gr.bullet(?, 9))
end;
```

or by using a type abbreviation in the signature:

```
functor Try(Gr:
  sig
    type G = int
    val e : G
    val bullet : G*G->G
    val inv : G->G
  end) =
struct
  val x = Gr.inv(Gr.bullet(?, 9))
end;
```
Since the above functors are all closed — in the sense that they contain no free identifiers apart from identifiers which are available initially (e.g., \textit{int} and \textit{+}) — it is possible to compile the functors. When a functor has been successfully compiled, one knows that the body of the functor is well-typed assuming only what the parameter signature reveals about the parameter. Thus the parameter signature is not merely a comment about what structures the functor needs; it is a guarantee that whenever one provides an actual structure that matches the parameter signature, one can combine the functor and the argument structure without violating the type soundness of the functor body.

The result signature in a functor declaration is optional. Also, in SML 96 one can choose between specifying the result signature with opaque and transparent matching. (SML 90 provides only transparent matching.)

6 Functor Application

The way one uses a functor is to apply it to an actual argument which matches the parameter signature, e.g.,

\begin{verbatim}
structure S = Try(Z)
\end{verbatim}

Hence combining modules is akin to combination (i.e., application) in the \(\lambda\)-calculus: a functor can be regarded as a map from structures to structures. In a functor application (\textit{Try}(Z)) it is first checked that the argument structure (Z) matches the parameter signature (\textit{GROUP}) of the functor (\textit{Try}). If the match fails, an error message is printed. Otherwise, the body of the functor is evaluated, resulting in a structure. In our example, this structure is then bound to a structure identifier (S) but that is not part of the functor application per se.

Type information is propagated through functor application. For example, consider the application

\begin{verbatim}
structure SqZ = Sq(Z);
\end{verbatim}

After the declaration we have \textit{SqZ.G} = \textit{int} * \textit{int}, obtained as the result of simplifying the declaration \textbf{type} \textit{G} = \textit{Gr}. \textit{G} * \textit{Gr}. \textit{G} (which is part of the body of \textit{Sq}), using that \textit{Gr} = Z.

If the functor has been declared using an opaque result signature, the result structure will only have the type equalities which are specified in the result signature. Thus the equality \textit{SqZ.G} = \textit{int} * \textit{int} would not hold if we had used \texttt{:> } instead of : in the declaration of \textit{Sq}. If one prefers using opaque signature constraints, one can retrieve the equality by imposing a \textbf{where type} qualification on the result signature when the functor is declared:
functor \( S(q(\text{Gr: GROUP}) \Rightarrow \text{GROUP}) \)

\[ \text{where} \quad \text{type} \ G = \text{Gr}.G \star \text{Gr}.G \]

\begin{verbatim}
struct
  type G = Gr.G * Gr.G
  val e = (Gr.e, Gr.e)
  fun bullet((a1, b1), (a2, b2)) =
    (Gr.bullet(a1, a2),
     Gr.bullet(b1, b2))
  fun inv(a, b) = (Gr.inv a, Gr.inv b)
end;
\end{verbatim}

7 Building Systems

Suppose we want to create a system consisting of three structures, \( A, B \) and \( C \), where \( B \) refers to \( A \) and \( C \) refers to both \( A \) and \( B \). The situation can be drawn as follows:

\[
\begin{array}{c}
  A \\
  \downarrow \\
  B \\
  \downarrow \\
  C
\end{array}
\]

Suppose that \( A, B \) and \( C \) have to match signatures \( \text{SIGA}, \text{SIGB} \) and \( \text{SIGC} \), respectively. The simplest way to construct the system is to have three structure declarations after each other:

\[
\begin{align*}
  \text{structure A:} & \quad \text{SIGA} = \text{strexp}_A; \\
  \text{structure B:} & \quad \text{SIGB} = \text{strexp}_B; \\
  \text{structure C:} & \quad \text{SIGC} = \text{strexp}_C;
\end{align*}
\]

where \( \text{strexp}_A, \text{strexp}_B \) and \( \text{strexp}_C \) are appropriate structure expressions, such that \( \text{strexp}_B \) contains free occurrences of qualified identifiers starting with \( A \) and \( \text{strexp}_C \) contains free occurrences of qualified identifiers starting with \( A \) or \( B \). However, this organisation does not give a clear picture of the dependencies between the three modules. (To see whether \( C \) depends on \( A \), one has to scan the entire declaration of \( C \).)

To make the dependencies explicit (and to facilitate separate compilation) one can use functors instead:
functor mkA() = strexp₄;

functor mkB(A: SIGA): SIGB = strexp₆;

functor mkC(
    structure A: SIGA
    structure B: SIGB; SIGC = strexp₈;

structure A = mkA();
structure B = mkB(A);
structure C = mkC(
    structure A = A
    structure B = B);

Incidentally, this example illustrates how one writes nullary functors and functors
with more than one structure parameter, in the latter case, one has to put the
keyword structure in front of each structure parameter, and this is repeated
when the functor is applied.

The signature SIGB may specify a type which really stems from SIGA (an
example will be given below). It may then be necessary for mkC to assume that
the two types A.t or B.t are actually the same type. For example, consider:\n
signature SIGA =
    sig
    type t
    val mk: int→ t
    val p: t→t→ t
    end;

signature SIGB =
    sig
    type b
    val b0: b
    type t
    val f: b→ t
    end;

\footnote{In SML 96, the example has to be modified slightly, since chr and ord are changing
type.}
signature SIGC =
  sig
  type t
  val test : t
 end;

functor mkA() : SIGA =
struct
  type t = string
  fun mk(i : int) : string =
      chr((i + ord "a") mod 128)
  fun p(n, m : string) = n \ m (* ^ means string concatenation *)
end;

functor mkB(A: SIGA) : SIGB =
struct
  type b = string
  val b0 = "abc"
  type t = A.t
  fun f(s : string) = A.mk(size s)
end;

functor mkC(
  structure A: SIGA
  structure B: SIGB) : SIGC =
struct
  type t = A.t
  val test = A.p(A.mk 16, A.p(A.mk 4, B.f(B.b0)))
end;

structure A = mkA();
structure B = mkB(A);
structure C = mkC(
  structure A = A
  structure B = B);

The declarations up to and including mkB are all fine; but the declaration of
mkC is ill-typed. Indeed, the ML Kit complains:

A.p(A.mk 16, A.p(A.mk 4, B.f(B.b0)))

********

Type clash,
operand suggests operator type: t * t->t
but I found operator type: t * t<162)->t
The mysterious $162$ in the last line indicates that the type differs from the corresponding type in the line above. Indeed, B.f(B.b0) has type B.t and A.mk 4 has type A.t and nowhere did we state that those two types be the same (as is required by the type of A.p). In some cases such a type error is an indication that the functor is wrong, i.e., that one has confused two types. But in this case, we really want to say that A.t and B.t are the same type, which we achieve by inserting a type sharing constraint in the start of mkC:

functor mkC(
  structure A: SIGA
  structure B: SIGB
  sharing type A.t = B.t : SIGC =
  ... as before ... 
)

After this correction, the declaration of mkC is well-typed. Moreover, the application of mkC is well-typed: it is automatically checked that the sharing constraint is satisfied, which it is with A.t = B.t = string. The result is a system which consists of three structures, as depicted earlier.

Exercise 3. What is the value of C.test?

The preceding examples (excluding the SML 96 examples) can be found in the file examples.sml. To run them, start an ML session in the same directory as the examples file. Then type: use "examples.sml";
PART 2: PRACTICAL

8 Implementing a Polymorphic Type-Checker

The purpose of this practical is to allow you to work through a slightly larger example of program development using ML modules. You are given a collection of modules that implement a type checker and interpreter for Mini ML, a tiny subset of the SML Core language.

The system can be executed and you can modify and extend it provided you have access to an implementation and to the files listed in Appendix B. We provide a parse functor which can parse a Mini ML source expression (represented as a string) into an abstract syntax tree. The rest of the interpreter works on abstract syntax trees. Unlike most real ML systems, the Mini ML system is an interpreted system. Your job will be to work on the polymorphic type checker.

Here is the grammar for Mini ML:

\[
\begin{align*}
exp & ::= exp + exp \\
& | exp - exp \\
& | exp \ast exp \\
& | true \\
& | false \\
& | exp = exp \\
& | if \ exp \ then \ exp \ else \ exp \\
& | exp :: exp \\
& | [ \ exp_1, \ldots, \ exp_n ] \ (n \geq 0) \\
& | let \ x = \ exp \ in \ exp \ end \\
& | let \ rec \ x = \ exp \ in \ exp \ end \\
& | x \\
& | fn \ x \Rightarrow \ exp \\
& | exp \ (\ exp \ ) \ (function \ application) \\
& | n \ (natural \ numbers) \\
& | (\ exp \ )
\end{align*}
\]

The abstract syntax of Mini ML is defined as a datatype in the signature `EXPRESSION`.

Exercise 4. Find and read this signature. What is the constructor corresponding to `let` expressions?

The interpreter uses a `typechecker` to check the validity of input expressions and an `evaluator` to evaluate them. Initially, the typechecker and evaluator handle only a tiny subset of Mini ML.

The typechecker and the evaluator can be developed independently as long as you do not change the signatures. The development of the typechecker and
the evaluator need not be in step. You can disable either by assigning false to one of the references tc and eval.

The source of the bare interpreter is in Appendix A. An overview of how to run the systems is provided in Appendix B.

Exercise 5. Find and read the signature of the interpreter (it is called INTERPRETER).

We program with signatures and functors only. After the signatures, which we shall not yet study, the first functor is the interpreter itself.

Exercise 6. Find this functor. Find the application of Ty.prType. Find it's type. What do you think Ty.prType is supposed to do? What is the type of abstrsyn? What do you think the evaluator is supposed to do when asked to evaluate something which has not yet been implemented?

We shall now describe Version 1, the bare typechecker, and then proceed to the extensions.

9 Version 1: The bare Typechecker

The first version is just able to type check integer constants and +. As signature TYPE reveals, the type Type of types is abstract (in the sense that the constructors are hidden), but there are functions we can use to build basic types and decompose them. unTypeInt is one of the latter; it is supposed to raise exception Type if applied to any Mini ML type different from the Mini ML integer type.\(^3\) This is a common way of hiding implementation details and it might be helpful to take a look at functor Type, which can produce a structure which matches the signature Type.\(^4\)

As revealed by the signature TYPECHECKER, the typechecker is going to depend on the abstract syntax and a Type structure. Notice that it is possible to specify structures in signatures as well as values and types.\(^5\) Similarly, it is possible to declare structures inside structures; such structures are called substructures.\(^6\) As you can see from the declaration of functor TypeChecker, all the typechecker knows about the implementation of types is what is specified by the signature TYPE. This allows us to experiment with the implementation of types to obtain greater efficiency without changing the typechecker, as we shall see in the later stages.

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\(^3\) In SML it is legal to use the same identifier as an exception constructor and a type constructor — the position of the identifier occurrence uniquely determines the identifier class.

\(^4\) It is also legal to use the same identifier as a signature identifier, a functor identifier and a structure identifier — the position of the identifier occurrence uniquely determines the identifier class.

\(^5\) However, it is not possible to specify functors or signatures in signatures.

\(^6\) However, it is not possible to declare functors or signatures inside structures.
Exercise 7. Functor TypeChecker is hostile to any expression which is not an integer constant or a sum expression. Modify the typechecker to handle \texttt{true}, \texttt{false}, and multiplication of integers. Make sure the revised functor compiles and runs. Assuming that your revised version of Appendix A is stored in file \texttt{myversion1.sml}, type:

```haskell
map use ["myversion1.sml", "parser.sml", "buildi.sml"];
```

Once the parser has been compiled once, you can omit it from the list. However, you have to compile the build file after each modification of your code, since the build file contains all the functor applications that build the system.

10 Version 2: Adding lists and polymorphism

The first extension is to implement the type checking of lists. In Version 1 the type of an expression could be inferred either directly (as in the case of \texttt{true} and \texttt{false}), or from the type of the subexpressions (as in the case of the arithmetic operations). When we introduce list, this is no longer the case. For example, consider the expression

```haskell
if (\texttt{[]} = [9]) then 5 else 7
```

Suppose we want to type check (\texttt{[]} = [9]) by first type checking the left subexpression \texttt{[]} , then the right subexpression [9] and finally checking that the left and right-hand sides are of the same type before returning the type \texttt{bool}. The problem now is that when we try to type check \texttt{[]} we cannot know that this empty list is supposed to be an integer list. The typechecker therefore just subcribes the type \texttt{'a list} to \texttt{[]} , where \texttt{'a} is a (Mini ML) type variable. The [9] of course turns out to be an \texttt{int list}. The typechecker now unifies the two types \texttt{'a list} and \texttt{int list} resulting in the substitution that maps \texttt{'a to int}. Hence the type of the expression \texttt{[]} depends not just on the expression itself, but also on the context of the expression. The context can force the type inferred for the expression to become more specific.

To implement all this, we first extend the \texttt{TYPE} signature and introduce a new signature, \texttt{UNIFY}, as shown in Figure 1.

The nice thing is that we can extend the typechecker without knowing anything about the inner workings of unification, simply by including a formal parameter of signature \texttt{UNIFY} in the typechecker functor. The complete functor is in the file \texttt{version2.sml}, but the most important bits are shown in Figure 2.

Here we see a new form of sharing constraint, namely sharing between structures. In SML 90 this specifies that when the functor is applied to actual structures \texttt{Ty} and \texttt{Unify}, it must be the case that \texttt{Ty} is the same substructure as the \texttt{Type}-substructure of \texttt{Unify}. This of course implies that types that are specified in both \texttt{Ty} and \texttt{Unify} are shared as well, e.g., we have the type equality \texttt{Ty.Type = Unify.Type.Type}. In SML 96, structure sharing has a weaker semantics: there is no notion of identity of structure; structure sharing constraints are still allowed, but they just abbreviate a sequence of type sharing constraints.
signature TYPE =
  sig
eqtype tyvar
  val freshTyvar: unit -> tyvar
  (*,... components omitted,...*)
  val mkTyypeTyvar: tyvar -> Type
    and unTypeTyvar: Type -> tyvar

  val mkTypeList: Type -> Type
    and unTypeList: Type -> Type
end

type subst
val id: subst
  (*the identity substitution;*)
val mkSubst: tyvar*Type -> subst
  (*make singleton substitution;*)
val on : subst* Type -> Type
  (*application*)

val prType: Type->string
  (*printing*)
end

signature UNIFY =
  sig
  structure Type: TYPE
  exception NotImplemented of string
  exception Unify
  val unify: Type.Type * Type.Type ->
    Type.subst
end;

Fig. 1. Signatures TYPE and UNIFY

We also have to extend the Type functor to meet the enriched TYPE signature, see Figure 3.

Exercise 8. Extend the typechecker of Version 2 to handle equality.
functor TypeChecker
  ((*,..,*)
structure Ty: TYPE
structure Unify: UNIFY
  sharing Unify.Type = Ty
)="
struct
  infix on
val (op on) = Ty.on
(*,..,*)

fun tc (exp: Ex.Expression) : Ty.Type =
  (case exp of
   (*,..,*)
   | Ex.LIST expr [] =>
     let val new = Ty.freshTyvar()
     in Ty.mkTypeList(
       Ty.mkTypeTyvar new)
     end
   | Ex.CONSEXPR (e1,e2) =>
     let
       val t1 = tc e1
       val t2 = tc e2
       val new = Ty.freshTyvar ()
       val newt = Ty.mkTypeTyvar new
       val t2' = Ty.mkTypeList newt
       val S1 =
         Unify.unify(t2, t2')
       handle Unify.Unify=>
         raise TypeError(e2,
           "expected list type")

       val S2 =
         Unify.unify(S1 on newt,
           S1 on t1)
       handle Unify.Unify=>
         raise TypeError(exp,
           "element and list have different types")
     in S2 on (S1 on t2)
     end
   )
   handle Unify.NotImplemented msg =>
     raise NotImplementedException
end; (*TypeChecker*)

Fig. 2. The TypeChecker functor
functor Type() : TYPE =
struct
  type tyvar = int
val freshTyvar =
  let val r = ref 0
  in fn() => (r := !r + 1; !r)
end

datatype Type = INT
| BOOL
| LIST of Type
| TYVAR of tyvar

fun mkTyvar tyvar = TYVAR tyvar
and unTyvar(TYVAR tyvar) = tyvar

| unTyvar _ = raise Type

fun mkList(t)=LIST t
and unList(LIST t) = t

| unList _ = raise Type


type subet = Type -> Type

fun Id x = x

fun mkSubst(te,ty) =
let
  fun su(TYVAR te') =
    if te=te' then ty
    else TYVAR te'
  | su(INT) = INT
  | su(BOOL)= BOOL
  | su(LIST ty') =
    LIST (su ty')
  in su
end

fun on(S,t) = S(t)

fun prType = (*...*)
end;

Fig.3. The Type functor
11 Version 3: A different implementation of types

Version 3 arises from Version 2 by replacing the Type functor by a different implementation of types. Instead of representing substitutions as functions, Version 3 implements type variables by references (pointers) so that it can perform substitutions very efficiently, by assignments. Here is an outline of the code:\footnote{\textsuperscript{7}}

```ocaml
functor ImpType() : TYPE =

  struct
  datatype 'a option =
    NONE
  | SOME of 'a
  datatype Type =
    INT | BOOL
  | LIST of Type
  | TYVAR of tyvar
  withtype tyvar =
    Type option ref
  fun freshTyvar() = ref (NONE)
  exception Type
  fun mkTypeInt() = INT
  and unType Int(INT) =()
  | (*...*)
  | unTypeInt(TYVAR(ref(SOME t))) =
  unType Int t
  | unTypeInt _ = raise Type
  (*...*)
  type subst = unit
  val Id = ();
  exception MkSubst;
  fun mkSubst(tv,ty)=
    case tv of
      ref(NONE) => tv := (SOME ty)
    | ref(SOME t) => raise MkSubst
  fun on(S,t)= t
  fun prType = (*...*)
end;
```

Exercise 9. You will find the prType operation in ImpType in Version 3 rather unsatisfactory; make modifications to correct this. (Hint: do not change anything but the functor.)

\footnote{\textsuperscript{7}} The withtype construct declares a type abbreviation within a datatype declaration.
12 Version 4: Introducing variables and let

We now extend Version 3 by implementing the type checking of let expressions and of identifiers.

The typechecker function tc now has to take two arguments,

$$tc(TE, e)$$

where e is an expression and TE is a type environment, which maps variables occurring free in e to type schemes. The definition of what a type scheme is will be given below; for now it suffices to know that every type can be regarded as a type scheme.

To take an example, if TE maps x to int and y to int, then tc will deduce the Mini ML type int for the expression x+y. However, if TE mapped y to bool, there would be a type error.

The fact that we can bind variables to expressions whose types have been inferred to contain type variables means that we get type variables in the type environment. For instance, to type check

```ml
let x = [] in 4 :: x end
```

we first check [] yielding the type 'a1 list, say. Then we bind x to the type scheme \( \forall 'a1.'a1 list \). Here the binding \( \forall 'a1 \) of 'a1 indicates that when we look up the the type of x in the type environment, we return a type obtained from the type scheme \( \forall 'a1.'a1 list \) by instantiating the bound variables (here just 'a1) by fresh type variables. In our example, when we look up x in the type environment during the checking of \( 4 :: x \), we instantiate \( 'a1 \) to a fresh type variable 'a2, say, yielding the type \( 'a2 list \) for x. Thus we get to unify int list against \( 'a2 list \), yielding the substitution of int for \( 'a2 \).

Throughout the body of the let, x will be bound to \( \forall 'a1.'a1 list \) in the type environment. Since we take a fresh instance of this type scheme each time we look up x, we can use x both as an int list and as an int list list, say:

```ml
let x = [] in  (4::x)::x end
```

Exercise 10. Assuming that you instantiate the bound 'a1 to 'a3 when you meet the last occurrence of x, what two types should be unified, and what is the resulting substitution on 'a3?

In ML, a type scheme always takes the form \( \forall \alpha_1 \ldots \alpha_n. \tau \), \( n \geq 0 \), where \( \alpha_1, \ldots, \alpha_n \) are type variables and \( \tau \) is a type not containing quantifiers. In the fragment of Mini ML considered so far, all type schemes inferred by the algorithm will be closed (i.e., any type variable occurring in \( \tau \) is amongst the \( \alpha_1, \ldots, \alpha_n \)), but when one introduces functions and application, this no longer is the case.

Exercise 11. Extend the type checker (Version 4) to handle conditionals and equality.
Exercise 12. For the extra keen. Extend Version 4 to cope with lambda abstraction ($\text{fn}$) and application. First, you have to introduce arrow types with constructors and destructors. Then you have to change the type of $\text{close}$ so that it takes two arguments, namely a type environment and a type. It should return the type scheme that is obtained by quantifying all the type variables that occur in the type but do not occur free in the type environment.

Then you can modify the type checker. When you type check a lambda abstraction, you just bind the formal parameter to the trivial type scheme which is just a fresh type variable (no quantified variables). Thus the type environment can now contain type schemes with free type variables.

An application $\text{tc}(\text{TE}, e)$ now yields two arguments, namely a type $t$ and a substitution $S$; the idea is that if you apply the substitution $S$ to the type environment $\text{TE}$, which now can contain free type variables, the expression $e$ has the type $t$. When an expression consists of more than one subexpression, the type environment gradually becomes more and more specific by applying the substitutions produced by the checking of the subexpressions one by one. Moreover, the substitution returned from the whole expression is the composition of these individual substitutions. (You have to extend the $\text{TYPE}$ signature (and the $\text{Type}$ functor) with composition of substitutions.

Finally, you can extend the unification algorithm to cope with arrow types. (This will also use composition of substitutions.)

Exercise 13. Finally, extend type type checker (Version 4) to handle recursive functions. In $\text{let rec} \ f = e_1 \ \text{in} \ e_2 \ \text{end}$, $e_1$ must be a lambda abstraction and the typing rule is

$$
\frac{TE + \{ f \mapsto \tau \} \vdash e_1 : \tau}{TE + \{ f \mapsto \text{close}(TE, \tau) \} \vdash e_2 : \tau'}
$$

13 Acknowledgements

The parser and evaluator are due to Nick Rothwell.

14 Further Reading

The Definition of Standard ML [MTH90] defines Standard ML formally. It is accompanied by a Commentary [MT94]. Milner's report on the Core Language [Mil84], MacQueen’s modules proposal [Mac84] and Harper’s I/O proposal were unified in [RHM86].

Several books on Computer Programming, using Standard ML as a programming language, are available [kW87, Rea89, Pau91, Sta92, CMP93]. In addition, there are medium-length introductions [Har86, Tof89].
Compilation techniques are treated by Appel[App92]. In this note we have used bits of The Edinburgh Standard ML Library[Ber91].

There is a large body of research papers related to ML, none of which we will cite on this occasion.

References

Appendix A: The bare Interpreter (Version 1)

(* interp1.sml: Mini ML interpreter, VERSION 1 *)

signature INTERPRETER=
  sig
    val interpret: string -> string
    val eval: bool ref
    and tc : bool ref
  end;

  (* syntax *)

signature EXPRESSION =
  sig
    datatype Expression =
      SUMexpr of Expression * Expression |
      DIFFexpr of Expression * Expression |
      PRODexpr of Expression * Expression |
      BOOLexpr of bool |
      EQexpr of Expression * Expression |
      CONDexpr of Expression * Expression * Expression |
      CONSexpr of Expression * Expression |
      LISTexpr of Expression list |
      DECLExpr of string * Expression * Expression |
      RECDECLexpr of string * Expression * Expression |
      IDENTexpr of string |
      LAMBDAexpr of string * Expression |
      APPEXpr of Expression * Expression |
      NUMBERexpr of int
  end

  (* parsing *)

signature PARSER =
  sig
    structure E: EXPRESSION
      exception Lexical of string
      exception Syntax of string

    val parse: string -> E.Expression
  end
(* environments *)

signature ENVIRONMENT =
  sig
    type 'object Environment
    exception Retrieve of string
    val emptyEnv: 'object Environment
    val declare: string * 'object * 'object Environment -> 'object Environment
    val retrieve: string * 'object Environment -> 'object
  end

(* evaluation *)

signature VALUE =
  sig
    type Value
    exception Value
    val mkValueNumber: int -> Value
    and unValueNumber: Value -> int
    val mkValueBool: bool -> Value
    and unValueBool: Value -> bool
    val ValueNil: Value
    val mkValueCons: Value * Value -> Value
    and unValueHead: Value -> Value
    and unValueTail: Value -> Value
    val eqValue: Value * Value -> bool
    val printValue: Value -> string
  end

signature EVALUATOR =
  sig
    structure Exp: EXPRESSION
    structure Val: VALUE
    exception Unimplemented
    val evaluate: Exp.Expression -> Val.Value
  end

  (* type checking *)
signature TYPE =
  sig
  type Type

(*constructors and destructors*)
  exception Type
  val mkTypeInt: unit -> Type
      and unTypeInt: Type -> unit

  val mkTypeBool: unit -> Type
      and unTypeBool: Type -> unit

  val prType: Type->string
end

signature TYPECHECKER =
  sig
    structure Exp: EXPRESSION
    structure Type: TYPE
    exception NotImplemented of string
    exception TypeError of Exp.Expression * string
    val typecheck: Exp.Expression -> Type.Type
  end;

  (* the interpreters*)

functor Interpreter
  (structure Ty: TYPE
   structure Value : VALUE
   structure Parser: PARSER
   structure TyCh: TYPECHECKER
   structure Evaluator:EVALUATOR
     sharing Parser.E = TyCh.Exp = Evaluator.Exp
     and TyCh.Type = Ty
     and Evaluator.Val = Value
  ): INTERPRETER=

struct
  val eval= ref false (* toggle for evaluation *)
  and tc = ref true (* toggle for type checking *)
  fun interpret(str)=
    let val abstsyn= Parser.parse str
      val typestr= if !tc then Ty.prType(TyCh.typecheck abstsyn)
else "(disabled)"
val valuestr= if 'eval then
  Value.printValue(Evaluator.evaluate abstsyn)
else "(disabled)"

in valuestr ^= "" : "" typestr
end
handle Evaluator.Unimplemented =>
  "Evaluator not fully implemented"
| TyCh.NotImplemented msg =>
  "Type Checker not fully implemented " ^ msg
| Value.Value => "Run-time error"
| Parser.Syntax msg => "Syntax Error: " ^ msg
| Parser.Lexical msg => "Lexical Error: " ^ msg
| TyCh.TypeError(_,msg) => "Type Error: " ^ msg
end;

(* the evaluator *)

functor Evaluator
(structure Expression: EXPRESSION
 structure Value: VALUE):EVALUATOR=

  struct
  structure Exp= Expression
  structure Val= Value
  exception Unimplemented

  local
  open Expression Value
  fun evaluate exp =
    case exp
    of BOOLexpr b => mkValueBool b
     | NUMBERRexpr i => mkValueNumber i
     | SUMexpr(e1, e2) =>
         let val e1' = evaluate e1
             val e2' = evaluate e2
         in
         mkValueNumber(unValueNumber e1' + unValueNumber e2')
     end
     | DIFFexpr(e1, e2) =>
         let val e1' = evaluate e1
             val e2' = evaluate e2
         in
         mkValueNumber(unValueNumber e1' - unValueNumber e2')
     end
  end
end


in
  mkValueNumber(unValueNumber e1'
    - unValueNumber e2')
end

| PRODexpr(e1, e2) =>
| let val e1' = evaluate e1
| val e2' = evaluate e2
| in
  mkValueNumber(unValueNumber e1'
    * unValueNumber e2')
end

| EQexpr _ => raise Unimplemented
| CONDexpr _ => raise Unimplemented
| CONSexpr _ => raise Unimplemented
| LISTexpr _ => raise Unimplemented
|DECLexpr _ => raise Unimplemented
| RECDDECLexpr _ => raise Unimplemented
| IDENTexpr _ => raise Unimplemented
| LAMBDALexpr _ => raise Unimplemented
| APPLexpr _ => raise Unimplemented

in
  val evaluate = evaluate
end
end;

(* the type checker *)

functor TypeChecker
  (structure Ex: EXPRESSION
   structure Ty: TYPE)=
struct
  structure Exp = Ex
  structure Type = Ty
exception NotImplemented of string
exception TypeError of Ex.Expression * string

fun tc (exp: Ex.Expression): Ty.Type =
  case exp of
    Ex.BOOLexpr b => raise NotImplemented "(bool const)
| Ex.NUMBERexpr _ => Ty.mkTypeInt()
| Ex.SUMexpr(e1,e2) => checkIntBin(e1,e2)
| Ex.DIFFexpr(e1,e2) => raise NotImplemented "(minus)"
and checkIntBin(e1,e2) =
let val t1 = tc e1
val _ = Ty.unTypeInt t1
    handle Ty.Type=> raise TypeError(e1,
        "expected int")
val t2 = tc e2
val _ = Ty.unTypeInt t2
    handle Ty.Type=> raise TypeError(e2,
        "expected int")
in Ty.mkTypeInt()
end;

val typecheck = tc
end; (*TypeChecker*)

(* the basics -- nullary functors *)

functor Type():TYPE =
struct
datatype Type = INT
    | BOOL

exception Type

fun mkTypeInt() = INT
and unTypeInt(INT)=()
    | unTypeInt(_)= raise Type

fun mkTypeBool() = BOOL
and unTypeBool(BOOL)=()
fun prType INT = "int"
| prType BOOL = "bool"
end;

functor Expression(): EXPRESSION =
  struct
  type 'a pair = 'a * 'a

  datatype Expression =
    SUMexpr of Expression pair |
    DIFFexpr of Expression pair |
    PRODexpr of Expression pair |
    BOOLexpr of bool |
    EDexpr of Expression pair |
    CONDexpr of Expression * Expression * Expression |
    CONNexpr of Expression pair |
    LISTexpr of Expression list |
    DECLexpr of string * Expression * Expression |
    RECDECLexpr of string * Expression * Expression |
    IDENTexpr of string |
    LAMBDAAexpr of string * Expression |
    APPLexpr of Expression * Expression |
    NUMBERexpr of int
  end;

functor Value(): VALUE =
  struct
  type 'a pair = 'a * 'a

  datatype Value = NUMBERvalue of int |
                  BOOLvalue of bool |
                  NILvalue |
                  CONNvalue of Value pair

  exception Value

  val mkValueNumber = NUMBERvalue
  val mkValueBool = BOOLvalue

  val ValueNil = NILvalue
  val mkValueCons = CONNvalue
fun unValueNumber(NUMBERvalue(i)) = i |
          unValueNumber(_) = raise Value

fun unValueBool(BOOLvalue(b)) = b |
          unValueBool(_) = raise Value

fun unValueHead(CONSvalue(c, _)) = c |
          unValueHead(_) = raise Value

fun unValueTail(CONSvalue(_, c)) = c |
          unValueTail(_) = raise Value

fun eqValue(c1, c2) = (c1 = c2)

(* Pretty-printing *)

fun intToString(i:int)= (if i<0 then "-" else ")"
          natToString (abs i)
and natToString(n:int)=
          let val d = n div 10 in
           if d = 0 then chr(ord"0" + n)
          else natToString(d) chr(ord"0" + (n mod 10))
          end
fun printValue(NUMBERvalue(i)) = intToString(i) |
          printValue(BOOLvalue(true)) = "true" |
          printValue(BOOLvalue(false)) = "false" |
          printValue(NILvalue) = "[]" |
          printValue(CONSvalue(c)) =
          "[" ^ printValueList(c) ^ "]"
and printValueList(hd, NILvalue) = printValue(hd) |
          printValueList(hd, CONSvalue(tl)) =
          printValue(hd) ^ "," ^ printValueList(tl) |
          printValueList(_) = raise Value
end;
Appendix B: Files

The following files are available:

Part 1

- examples.sml  The examples from Part 1

Part 2

- interp1.sml  Version 1 (as included in Appendix A).
- interp2.sml · interp4.sml  The other versions.
- build1.sml  the structure declarations needed to build Version 1.
- build2.sml · build4.sml  Similarly for the other versions.
- parser.sml  The parser functor.

To build Version 3, say, you type the following (assuming you have copied the files to your directory):

    use "interp3.sml";
    use "parser.sml";
    use "build3.sml";

Since the parser functor is completely closed, you don’t have to include it more than once in every session, although you will probably want to build your system several times while you experiment with the extensions.

This article was processed using the \texttt{I\STeX} macro package with LLNCS style