1 ML at a Glance

Suppose we were to draw a map of the landscape of programming languages. Where would ML fit in? COBOL and ML could safely be put down far apart. The input/output facilities in COBOL operate on specific kinds of input/output devices, for instance allowing the programmer to declare index sequential files. ML just has the notion of STREAMS, a stream being a sequence of characters, much like streams in UNIX or text files in PASCAL. On the other hand, ML is extremely concise compared to the verbose COBOL and ML is much better suited for structuring data and algorithms than COBOL is.

ML is closer related to PASCAL. Like PASCAL, ML has data types and there is a type checker which checks the validity of programs before they are run. Both PASCAL and ML follow the tradition of ALGOL in that variables can have local scope which is determined statically from the source program. However, PASCAL and ML are radically different in how algorithms are expressed. In PASCAL, as in many other languages, a variable can be updated (using :=). Algorithms are often expressed as iterated sequences of statements (using while loops, for instance), where the effect of executing one statement is to change the underlying store. In ML, statements are replaced by EXPRESSIONS; the effect of evaluating an expression is to produce a value. Moreover, variables cannot be updated; REFERENCES are special values that can be updated, and as all other values they can be bound to identifiers, but only rarely are the values one binds to variables references. Iteration is expressed using recursive functions instead of loops. In ML, functions are values which can be passed as arguments to functions and returned as results from functions, and ML programmers do this all the time. ML is an example of a FUNCTIONAL language; PASCAL is an example of a PROCEDURAL language.

LISP is also sometimes referred to as a functional language. In LISP, programs can be treated as data, so that LISP programs directly can decompose and transform LISP programs. This is harder in ML. On the other hand, the type discipline of ML is extremely helpful in detecting many of the mistakes that pass unnoticed in a LISP program.

Like ADA, ML has language constructs for writing large programs. Roughly speaking, a STRUCTURE in ML corresponds to a PACKAGE in ADA; a SIGNATURE corresponds to a PACKAGE INTERFACE and a FUNCTOR in ML corresponds to a GENERIC PACKAGE in ADA. However, ML admits structures (not just types) as parameters to functors.

1.1 An ML session

An ML session is an interactive dialogue between the ML system and the user. You type a PROGRAM in the form of one or more DECLARATIONS (terminated by semicolon) and the system responds either by accepting the declarations or, in case the program is ill-formed, by printing an error message.

To give a concrete idea about what ML programs look like, we shall work through the following example. Consider the problem of implementing heaps. A HEAP is a binary tree of ITEMS, for example:

```
    7
   /\  
 11 9
 /  /  
17 15
```


For a binary tree to be a heap, it must satisfy that for every item \( i \) in the tree, \( i \) is less than or equal to all items occurring below \( i \). In the above picture items are integers and the relation “less than or equal” is the normal \( \leq \) on integers. The advantage of a heap is that it always gives fast access to a minimal item and that it is easy to insert and delete items from a heap. This has made the heap a popular data structure in a number of very different applications. It was originally conceived under the name “priority queue” as a means of scheduling processes in an operating system; in that case the items are processes and the partial ordering is that process \( p \) is less than or equal to process \( q \), if \( p \) should be executed no later than \( q \). Heaps are also used in the heap sort algorithm, which is based on the observation that one can sort a list of items by first inserting the items one by one in a heap and then removing them one by one.

1.2 Types and Values

In the following figures we present the ML declarations the author provided in this particular session. The responses from the ML compiler are not shown. For clarity, the actual input has been edited using typewriter font for the reserved words and italics for identifiers, regardless of whether these identifiers are pervasives (e.g. \( \text{int} \)) or declared by the user (e.g. \( \text{item} \)).

```ml
type item = int;
fun leq(p: item, q: item): bool =
    p <= q;
infix leq;
fun max(p, q) = if p leq q then q else p
and min(p, q) = if p leq q then p else q
datatype tree = L of item
    | N of item * tree * tree;
val t = N(7, L 11, N(9, L 17, L 15));
fun top(L i) = i
    | top(N(i, _, _)) = i;
```

We start out by considering integer heaps only; therefore we first declare the type \( \text{item} \) to be an abbreviation for \( \text{int} \). Then we declare a function \( \text{leq} \) to be the pervasive \( \leq \) on integers. We then declare that \( \text{leq} \) is to be used as an infix operator, as illustrated in the declaration of the two functions \( \text{max} \) and \( \text{min} \).

Every binary tree is either a leaf containing an item or it is a node containing an item and two trees (the subtrees). This is expressed by the \( \text{datatype} \) declaration. \( \text{datatype} \) declarations are automatically recursive, i.e. data types can be declared in terms of themselves. This is illustrated by the declaration of \( \text{tree} \). This data type has two \( \text{constructors} \), \( L \) and \( N \). Note that for example 7 is an item, but \( L \text{ applied to } 7 \), written \( L(7) \), or just \( L \ 7 \), is of type \( \text{tree} \). Then the heap from the earlier picture is bound to the value variable \( t \).

**Exercise 1** Declare a heap \( t' \) of the same depth as \( t \) containing the integers 78, 34, 5,
12, 15, 28, and 9.

To define a function on trees it will suffice to define its value in the case the argument tree is a node and in the case the tree is a node. The declaration of the function top illustrates this. (top applied to a tree returns the item at the top of the tree). (L i) and (N(i, _, _)) are examples of patterns. Applying a function (here top) to an argument (e.g. t) is done by matching the argument against the patterns till a matching pattern is found. For example top t evaluates to 7.

### 1.3 Recursive Functions

```ml
fun depth(L _) = 1 | depth(N(i, l, r)) = 1 + max(depth l, depth r);
depth t;

fun isHeap(L _) : bool = true | isHeap(N(i, l, r)) = i leq top l andalso i leq top r andalso isHeap l andalso isHeap r
```

The function depth maps trees to integers; for instance depth t evaluates to 3. As spelled out in the declaration of depth, the depth of a leaf is 1 and the depth of any other tree is 1 plus the maximum of the depths of the left and right subtrees. The function depth is RECURSIVE, i.e. defined in terms of itself. Another example of a recursive function is the function isHeap which when applied to a tree returns the value true if the tree is a heap and false otherwise.

### Exercise 2

Write a function size which when applied to a tree returns the total number of items in the tree.

### Exercise 3

The function top returns a minimal item of a heap. Write a recursive function maxItem which returns a maximal item.

### 1.4 Raising Exceptions

One often wants to define a function that cannot return a result for some of its argument values. Suppose, for example, that we wish to define a function initHeap which for given integer n returns a heap of depth n. This only makes sense for \( n \geq 1 \). This can be expressed in ML by raising an EXCEPTION in the case \( n < 1 \). The effect of evaluating the expression raise e, where e is an exception, is to discontinue the current evaluation. Often, the exception will be HANDLED by a handle expression (not illustrated by our examples); if no handler catches the exception, it propagates to the top-level where it will be reported as an uncaught exception.

```ml
val initial = 0
exception InitHeap
fun initHeap n =
  if n<1 then raise InitHeap
  else if n = 1 then L(initial)
  else let val t = initHeap(n - 1)
         in N(initial, t, t)
         end
```

Notice the let dec in exp end expression. To evaluate it, one first evaluates initHeap(n - 1) and binds the resulting value to t. Then one evaluates the body, N(initial, t, t) using this value for t. Notice that the scope of the declaration of t is the expression
\(N(\text{initial}, t, t); \text{in particular the two occurrences of } t \text{ in that expression do not refer to } N(7, L_{11}, N(9, L_{17}, L_{15})).\)

**Exercise 4** Define functions \(\text{leftSub}\) and \(\text{rightSub}\) which when applied to a tree returns the left and the right subtree, respectively.

Finally, we shall write a function \(\text{replace}\) which when applied to a pair \((i, h)\), where \(i\) is an item and \(h\) is a heap, returns a pair \((i', h')\), where \(i'\) is the item at the top of the heap \(h\) and \(h'\) is a heap obtained from \(h\) by inserting \(i\) in place of the top of \(h\). We must make sure that the resulting tree really is a heap. Therefore, in the case that \(i\) is to be inserted in a node above a subtree with a smaller item, \(i\) swops place with the smaller item.

\[
\text{fun replace}(i, h) = (\text{top } h, \text{insert}(i, h))
\]

\[
\text{and insert}(i, L \_ ) = L(i)
\]

\[
| \text{insert}(i, N(\_ , l, r)) =
\]

\[
| \quad \text{if } i \text{ leq min(top } l, \text{ top } r) \text{ then } N(i, l, r)
\]

\[
| \quad \text{else if (top } l \text{) leq (top } r) \text{ then } N(\text{top } l, \text{insert}(i, l), r)
\]

\[
| \quad \text{else (* top } r < \text{min(i, top } l) *)
\]

\[
| \quad N(\text{top } r, l, \text{insert}(i, r));
\]

\[
\text{val (out1, t1)} = \text{replace}(10, t);
\]

\[
\text{t};
\]

\[
\text{val (out2, t2)} = \text{replace}(20, t1)
\]

The special parenthesis (* and *) delimit comments.

**Exercise 5** In the case where one recursively inserts \(i\) in the left subtree, how can one be sure that it is valid to put \(\text{top } l\) above \(r\) in the tree?

If one types an expression followed by a semicolon (such as \(t;\) in the above program) the ML system evaluates the expression and prints the result. In the above example, it will turn out that even after we have “replaced” 7 by 10, \(t\) is bound to the original heap. Indeed, this “replacement” in no way affects the value bound to \(t\); it simply results in a new value, which subsequently is bound to \(t1\).

**Exercise 6** After the last declaration, what values are bound to \(\text{out2}\) and \(t2\)?

### 1.5 Structures

The above declarations of heaps and operations on heaps belong together. In ML there is a program unit called a **structure** which encapsulates a sequence of declarations. The following declaration declares a structure \(\text{Heap}\) containing all the declarations (copied from above) encapsulated by **struct** and **end**.
structure Heap =
struct
  type item = int;
  fun leq(p: item; q: item): bool = p <= q;
  fun max(p, q) = ...
  and min(p, q) = ...
datatype tree = L of item |
  N of item * tree * tree;
val t = ...
fun top(L i) = ...
fun depth(L _) = ...
fun isHeap(L _): bool = ...
val initial = 0
exception InitHeap
fun initHeap n = ...
fun replace(i, h) = ...
  and insert(i, L _) = ...
end; (* Heap *)

val smallHeap = Heap. initHeap(1);
Heap. replace(20, smallHeap);

signature HEAP =
sig
  type item
  val leq: item * item -> bool
  val max: item * item -> item
  val min: item * item -> item
  datatype tree = L of item |
    N of item * tree * tree
  val t: item
  val top: tree -> item
  val depth: tree -> int
  val isHeap: tree -> bool
  val initial: item
  exception InitHeap
  val initHeap: int -> tree
  val replace: item * tree -> item * tree
  val insert: item * tree -> tree
end; (* HEAP *)

As one can check, the structure Heap matches signature HEAP in the following sense: for every type specified in HEAP, there is a corresponding type in Heap; for every exception specified in HEAP, there is a corresponding exception in Heap; and for every value specified in HEAP there is a corresponding value in Heap which has the specified type.

1.7 Coersive Signature Matching

However, the signature HEAP reflects details of the implementation in Heap which heap users should not have to worry about. (Obviously, the value t is completely unnecessary, and there is no reason why users should have access to the constructors L and N given that we have already given the user initHeap and replace.) By pruning the signature we obtain...
the following shorter declaration of \textit{HEAP}.

\begin{verbatim}

signature \textit{HEAP} =
  sig
    type \textit{item}
    val \textit{leq}: \textit{item} \times \textit{item} \rightarrow \textit{bool}

    type \textit{tree}
    val \textit{top}: \textit{tree} \rightarrow \textit{item}
  exception \textit{InitHeap}
    val \textit{replace}: \textit{item} \times \textit{tree} \rightarrow \textit{item} \times \textit{tree}
end; (* \textit{HEAP} *)
\end{verbatim}

This is a much cleaner interface, so whenever we refer to \textit{HEAP} in the following, we mean this version.

In practice, one should write down a signature \textit{before} one attempts to write down a structure which matches it. In this way one can decide what types and operations are needed without having to think about algorithms at the same time. So let us assume that we started out by declaring the \textit{HEAP} signature. We then imprint the view provided by \textit{HEAP} on the declaration of the structure \textit{Heap} by a \textsc{signature constraint}:

\begin{verbatim}

structure \textit{Heap}: \textit{HEAP} =
  struct
    type \textit{item} = \textit{int};
    ...
  end; (* \textit{Heap} *)

? \textit{Heap}.\textit{t}
\textit{Heap}.\textit{replace}(7, \textit{Heap}.\textit{initHeap} 3);
\end{verbatim}

After this declaration of \textit{Heap}, we cannot write \textit{Heap}.\textit{t}, since \textit{t} is not mentioned in \textit{HEAP}. However, we can write \textit{Heap}.\textit{replace} as \textit{replace} is specified. Moreover, although \textit{HEAP} does not specify that \textit{item} should be \textit{int}, the ML system discovers that \textit{item} is in fact \textit{int} in \textit{Heap} and that is why 7 will be accepted as an \textit{item} in the application \textit{Heap}.\textit{replace}(7, \textit{Heap}.\textit{initHeap} 3). Thus a signature constraint may hide components of a structure, but it does not hide the true identity of the types declared in the structure, except that one can hide the constructors of a \texttt{datatype} by specifying it as a \texttt{type}.

\subsection*{1.8 Functor Declaration}

Almost all of what we did for heaps containing integer items would work for a heap whose items are of a different type. More precisely, given any type \textit{item}, any binary function \textit{leq} on items and any \textit{initial} item, the signature \textit{HEAP} is satisfied by the declarations we have already written. Let us specify the general requirements of a \textit{Heap} structure.

\begin{verbatim}

signature \textit{ITEM} =
  sig
    type \textit{item}
    val \textit{leq}: \textit{item} \times \textit{item} \rightarrow \textit{bool}
    val \textit{initial}: \textit{item}
end;
\end{verbatim}

What we are after is a structure which is parameterised on any structure, \textit{Item}, say, which matches \textit{ITEM}. In ML, a parameterised structure is called a \texttt{FUNCTOR}. The following table contains the complete functor declaration; the new bits are in bold face.
functor Heap(Item: ITEM): HEAP =
struct
  type item = Item.item
  fun leq(p: item, q: item): bool = Item.leq(p,q)
  fun intmax(i: int, j) = if i <= j then i else j
infix leq;
  fun max(p, q) = if p leq q then q else p
  and min(p, q) = if p leq q then p else q
datatype tree = L of item |
  N of item * tree * tree;
  fun top(L i) = i |
    top(N(i, _, _)) = i;
  fun depth(L _) = 1 |
    depth(N(i, l, r)) = 1 + intmax(depth l, depth r);
  fun isHeap(L _): bool = true |
    isHeap(N(i, l, r)) = i leq top l andalso 
      i leq top r andalso 
      isHeap l andalso 
      isHeap r
exception InitHeap
  fun initHeap n = 
    if n<1 then raise InitHeap 
    else if n = 1 then L(Item.initial) 
    else let val t = initHeap(n - 1) 
        in N(Item.initial, t, t) 
        end
  fun replace(i, h) = (top h, insert(i, h))
  and insert(i, L _) = L(i) |
    insert(i, N(_, l, r))= 
      if i leq min(top l, top r)
        then N(i, l, r)
      else if (top l) leq (top r) then 
        N(top l, insert(i, l), r)
      else (* top r < min(i, top l) *) 
        N(top r, l, insert(i, r));
end; (* Heap *)

In the first line, Item is the PARAMETER structure of the functor and HEAP is the RESULT signature of the functor. The BODY of the functor is everything after the = in the first line.

Notice that we included declarations of item and leq in the body of the functor; since the result signature specifies them, they must be provided. If you read the body carefully, you will see that it makes sense for any structure which matches ITEM.

Exercise 7 Declare a functor Pair which takes as a parameter a structure matching the simple signature

    sig type coord end

and has the following result signature:

    sig
      type point
    val mkPoint: coord * coord -> point
    val x_coord: point -> coord 
    val y_coord: point -> coord
    end

You do not have to name these signatures (by the use of signature declarations); they can be written down directly where you need them, if you prefer.

Exercise 8 When the author first tried to write the Heap functor, he simply copied the original depth function which used max, not intmax. However, the type checker did not let him get away with that. Why?

1.9 Functor Application

We can now get various heaps (indeed heaps of heaps) by applying the Heap functor to different argument structures. Of course, we can only apply it to structures that match ITEM; this will be checked by the compiler.

Here is how one can get a string heap:
structure StringItem =
struct
  type item = string
  fun leq(i:item, j) =
    ord(i) <= ord(j)
  val initial = " "
end;

structure StringHeap = Heap(StringItem)

val (out1, t1) =
  StringHeap.replace("abe",
    StringHeap.initHeap(1));
val (out2, t2) =
  StringHeap.replace("man", t1);

The pervasive ord function applied to a string $s$ returns the ASCII ordinal value of the first character in $s$, and raises exception Ord when $s$ is empty.

**Exercise 9** Declare a structure IntItem using the declarations we originally used for integer heaps. Then obtain a structure IntHeap by functor application.

**Exercise 10** How does one get an integer heap whose top is always maximal?

**Exercise 11** Declare a structure IntHeapHeap whose items themselves are integer heaps. (You can use the top function to define a leq function on integer heaps.)

### 1.10 Summary

ML consists of a core language and a modules language. The core language has values (functions are values), data types, type abbreviations and exceptions. The modules language has structures, signatures and functors. There is no actual language construct called a module, but ML programmers often refer to a module meaning “a structure or a functor”.

2 Programming with ML Modules

2.1 Introduction

This lecture gives a more thorough introduction to the modules part of ML and describes a methodology for programming with its main constructs: structures, signatures and functors.

The core language is interactive: you type a declaration, get a reply, type another declaration and so on, thus gradually adding more and more bindings to the top-level environment. If we could think strictly bottom-up, declaring one value or type in terms of the preceding values and types, without ever making unfortunate implementation decisions or losing the perspective of the entire project, then this gradual expansion of the top-level environment would be quite sufficient. Unfortunately, we cannot, indeed a program which is written as one long list of core language declarations can easily end up looking rather like a long shopping list where items have been added in the order they came to mind.

Regardless of whether a programming language is interactive or not, one needs the ability to divide large programs into relatively independent units which can be written, read, compiled and changed in relative isolation from each other.

One approach, taken by some, is to provide more or less language independent software packages that help programmers organise collections of programs typically by allowing (or forcing) them to document their programs in specific ways. The crucial problem with this approach is of course to ensure consistency between the documentation and the programs, in particular to ensure that the information held by the tool really is sufficient to ensure that the constituent units can be put together in a consistent manner.

Another approach, taken in several programming languages (e.g. Ada and ML), is to provide module facilities in the programming language itself. Many of the operations one needs when programming with modules are similar to operations one needs when programming in the small, so many ideas from usual programming languages apply to programming in the large as well. For instance, just as it is a type error (in the small) to add `true` and 7, say, so it is a type error (in the large) to write a module `M2`, say, assuming the existence of a module `M1` which provides a function `f`, and then combine `M2` with an actual module `M1` which either does not provide any `f` or provides an `f` of the wrong type. The idea is that such mistakes should be detected by a type checker at the modules level.

This leads to the exciting idea of having just one language with constructs that work uniformly for “small” as well as for “large” programs. One such language is Pebble by Burstall and Lampson. In Pebble records can contain types, so a module consisting of a collection of types and values is now itself a value, which for example can be passed as an argument to a function. There are some trade-offs, however. The ML type checker is based on a strict separation of run-time and compile-time. In designing the modules language it has been necessary to restrict the operations on types in comparison with the operations on values in order to maintain the static type checking. This has led to a stratified language, in which the modules language contains phrases from the core language, but not the other way around.

I shall use the term “module” rather vaguely to mean “a relatively independent program unit”. In particular languages they have been called “packages”, “clusters”,...
"modules" and in ML we use the word “structure”.

Likewise, there is no standard terminology for “the type of a module”, which has acquired names such as “package description”, “interface” and the ML term “signature”.

As we shall see, the real power of a modules system comes from the ability to parameterise modules. Ada has “generic packages”. ML has “functors”.

In ML, a structure is a collection of data types, types, values, exceptions and even other structures. A signature specifies types and data types and gives the types of values and exceptions. A functor is essentially a function from structures to structures. Functors cannot take functors as arguments, nor can they produce functors as results. The purpose of this lecture is to convince you that even this apparently simple notion of functor constitutes a powerful extension of the core language. As will be demonstrated, one can write an entire system using just signatures and functors and then build the system using functor applications.

Imagine we want to write a parser for a programming language. In order to build the parser top-down, we might start by sketching the parser itself. However, as programming normally is a complex process involving both the odd low-level implementation idea and more high-level considerations about overall structure, let us start at an intermediate level, the problem of writing a symbol table. (A symbol table is simply a facility which allows one to store and retrieve information about symbols.)

### 2.2 Signatures

The way one in ML sketches a structure is to write down a signature. Here is a first sketch of a symbol table signature, called $OTable$ as it is opaque in the sense that it does not reveal many implementation details.

```
signature $OTable$ =
  sig
    type table
    exception Lookup
    val lookup: table * Sym.sym -> Val.value
    val update: table * Sym.sym * Val.value
      -> table
  end
```

At this early stage, we cannot know exactly what symbols are going to be; nor can we know what kind of values we are going to store with the symbols. Therefore we imagine structures $Sym$ and $Val$ which declare the types $sym$ and $value$, respectively. $Sym.sym$ is an example of a LONG IDENTIFIER, in this case a long type constructor. The two structure identifiers $Sym$ and $Val$ are FREE in $OTable$.

There are many different ways of implementing a symbol table which matches this signature. One possibility is to use an association list, i.e. a list of pairs of symbols and values. Since the symbol table is going to be used extensively, we will probably want something more efficient. We cannot use an array, for arrays map integers (rather than symbols) to values. (Actually, the ML language definition does not include arrays, but they are provided in most implementations). But we can implement the symbol table as a hash table: we can require that the $Sym$ structure provides a hash function from symbols to integers and then assume the existence of another structure, $IntMap$, which implements maps on the integers. Since the hash function may map different symbols to the same integer, we take an $IntMap$ which maps integers
to lists of pairs of symbols and values:

signature TTable =
sig
datatype table = TBL of
  (Sym.sym * Val.value) list IntMap.map
exception Lookup
val lookup: table * Sym.sym -> Val.value
val update: table * Sym.sym * Val.value -> table
end

2.3 Structures

Here is a structure which implements a symbol table.

structure SymTbl =
struct
datatype table = TBL of
  (Sym.sym * Val.value) list IntMap.map
exception Lookup

fun find(sym,[[]]) = raise Lookup
| find(sym,(sym',v)::rest) =
  if sym = sym' then v
  else find(sym,rest)

fun lookup(TBL map, s) =
  let val n = Sym.hash(s)
    val l = IntMap.apply(map,n)
  in find(s,l)
  end
  end handle IntMap.NotFound => raise Lookup
  ...
end

When binding a structure to a structure identifier one can impose a signature constraint on the structure.

structure SymTbl : OTable =
struct
  ...
end

As a result, all identifiers of the structure that are not mentioned in the signature are hidden. In the above example we hide the constructor TBL and the function find. Besides update, the ... in SymTbl may declare extra values, exceptions and types, but as a result of the signature constraint, none of these extra components will be visible from outside the structure.

It is often the case that there is not a single signature which “best” constrains a given structure because different parts of the program should see different degrees of details of the structure. (The parser should be written using a opaque signature for the symbol table; by contrast, a structure which prints out the symbol table (for testing, for example) will need to know more details).

During the design of ML it was decided that it is vital to admit different views of the same structure. One way of achieving this is to bind the structure to more that one structure identifier, each time using a different signature constraint.

structure SymTbl : TTable =
struct
  datatype table = TBL of
    (Sym.sym * Val.value) list IntMap.map
  exception Lookup

fun find(sym, []) = raise Lookup
  | find(sym, (sym', v)::rest) = 
      if sym = sym' then v
      else find(sym, rest)

fun lookup(TBL map, s) = 
  let val n = Sym.hash(s)
      val l = IntMap.apply(map, n)
      in find(s, l)
  end handle IntMap.NotFound => raise Lookup

structure SmallTbl: OTable = SymTbl

The dynamic evaluation of the \texttt{struct...end} yields an environment just as if we had typed the constituent declarations at top-level. Dynamically, there is just one \texttt{lookup} function, for example, and as a result of the above declarations, this function is shared between \texttt{SymTbl} and \texttt{SmallTbl}.

Statically, however, the elaboration of the above declarations yields two different views of this environment. Since there is just one \texttt{lookup} function, we should of course be free to refer to it as \texttt{SymTbl.lookup} or \texttt{SmallTbl.lookup}, whichever we prefer. This requires that these two long identifiers have the same type; so the static semantics must be such that the types \texttt{SymTbl.table} and \texttt{SmallTbl.table} are considered shared.

### 2.4 Functors

Unfortunately, neither the declaration of the signature \texttt{OTable} nor the declaration of the structure \texttt{SymTbl} makes sense on its own. The reason is that they both contain free identifiers. \texttt{OTable} relies on structures \texttt{Val} and \texttt{Sym} and \texttt{SymTbl} in addition relies on \texttt{IntMap}. As a consequence, we can compile neither \texttt{OTable} nor \texttt{SymTbl} in the initial top-level environment.

What we need to achieve this is clearly the ability to abstract both \texttt{OTable} and \texttt{SymTbl} on their free identifiers. Such an abstraction is called a \texttt{FUNCTOR} in ML:

```ml
functor SymTblFct(
  structure IntMap: IntMapSig
  structure Val: ValSig
  structure Sym: SymSig)
  : sig
    type table
    exception Lookup
    val lookup: table * Sym.sym -> Val.value
    val update: table * Sym.sym * Val.value -> table
  end

struct
datatype table = TBL of (Sym.sym * Val.value) list
  IntMap.map
  exception Lookup

fun find(sym, []) = raise Lookup
  | find(sym, (sym', v)::rest) = 
      if sym = sym' then v
      else find(sym, rest)

fun lookup(TBL map, s) = 
  let val n = Sym.hash(s)
      val l = IntMap.apply(map, n)
      in find(s, l)
  end handle IntMap.NotFound => raise Lookup

...
end
```
Now the Sym, Val and IntMap that occur in the result signature and in the functor body are bound as formal parameters of the functor. Of course for this functor declaration to make sense, we must first declare the three signatures IntMapSig, ValSig and SymSig, but that can be done without worrying about how we get the corresponding structures. Indeed, declaring these signatures is healthy exercise, as it makes us summarise what a symbol table needs to know about symbols, values and intmaps.

Exercise 12 Declare the signatures IntMapSig, ValSig and SymSig. Also complete the functor body by declaring update, extending your signatures, if needed.

When, in due course, we have defined actual structures FastIntMap, Data and Identifier, say, corresponding to the formal structures IntMap, Val and Sym, respectively, we can obtain a particular symbol table by applying the symbol table functor.

```
structure MyTbl = SymTblFct(structure IntMap = FastIntMap
structure Val = Data
structure Sym = Identifier)
```

Dynamically, the functor body is not evaluated when the functor is declared, but once for each time the functor is applied. (In this respect, functors behave a functions in the core language.)

As part of the functor application, the compiler will check to see whether the actual argument structures match the specified signatures. If that is not the case (for instance, if Identifier does not contain a type sym or FastIntMap . apply takes three instead of two arguments) then an error will be reported and hence we are prevented from putting together the inconsistent structures.

What is the signature of MyTbl? It is not simply the result signature of SymTblFct, for that signature refers to the formal functor parameters Val and Sym. Clearly, if Identifier . sym is string and Data . value is real, then we should be able to write for instance

```
sqrt(MyTbl . lookup(t, “pi”))
```

The signature of MyTbl, therefore, is obtained by substituting the types of the actual arguments for the types in the formal result signature of SymTblFct.

```
sig
type table
exception Lookup
val lookup: table * Identifier . sym -> Data . value
val update: table * Identifier . sym * Data . value -> table
end
```

2.5 Substructures

As we saw above, the signature of the result of a functor application depends on the actual arguments to the functor. So apparently there is no single signature which describes all the symbol tables which can be created by applying SymTblFct. But how, then, are we going to declare a functor ParseFct, say, which we can apply to any symbol table created by SymTblFct?
The solution to this problem is to make explicit in the symbol table signature that any symbol table depends on a Val and a Sym structure.

signature SymTblSig =
structure Val: ValSig
structure Sym: SymSig
type table
val lookup: table * Sym.sym -> Val.value
val update: table * Sym.sym * Val.value 
-> table
end

datatype table = TBL of
(Sym.sym * Val.value)list IntMap.map
exception Lookup

fun find(sym,[]) = raise Lookup
| find(sym,(sym',v)::rest) =
  if sym = sym' then v
  else find(sym,rest)

fun lookup(TBL map, s) =
  let val n = Sym.hash(s)
  val l = IntMap.apply(map,n)
  in find(s,l)
  end handle IntMap.NotFound =>
    raise Lookup
...
end

The specifications of Val and Sym no longer refer to particular structures outside the signature, i.e. Val and Sym are now considered bound in the signature. (Of course the signature identifiers SymSig and ValSig are still free, but those you have already declared in the exercise.)

The idea is that a structures can contain not just values, exceptions and types as components, but even other structures. These are called the SUBSTRUCTURES of the structure. To make the result of SymTblFct match SymTblSig, we have to declare structures Val and Sym in the body. But that is easily done; we simply bind them to the formal parameters.

The specifications of Val and Sym no longer refer to particular structures outside the signature, i.e. Val and Sym are now considered bound in the signature. (Of course the signature identifiers SymSig and ValSig are still free, but those you have already declared in the exercise.)

The idea is that a structures can contain not just values, exceptions and types as components, but even other structures. These are called the SUBSTRUCTURES of the structure. To make the result of SymTblFct match SymTblSig, we have to declare structures Val and Sym in the body. But that is easily done; we simply bind them to the formal parameters.

functor SymTblFctK(  
structure IntMap: IntMapSig
structure Val: ValSig
structure Sym: SymSig): SymTblSig =
struct
structure Val = Val
structure Sym = Sym
functor SymTblFctK(  
structure IntMap: IntMapSig
structure Val: ValSig
structure Sym: SymSig): SymTblSig =
struct
structure Val = Val
structure Sym = Sym

2.6 Sharing
A signature for lexical analysers might be as follows (a lexical analyser reads individual characters from an input file and assembles them into symbols — in the case of a programming language typically reserved words and identifiers):

signature LexSig =
sig
structure Sym : SymSig
val getsym : unit -> Sym.sym
end

We have included the specification of a substructure Sym because a lexical analyser needs to know about symbols. (Indeed, if we want to declare LexSig before defining any particular Sym structure, we are forced to include the substructure specification.)
functor ParseFct(
  structure SymTbl: SymTblSig
  structure Lex: LexSig) =
struct
  let val next = Lex.getsym()
  in SymTbl.update(table, next, “declared”) end
end

Here is a first attempt at defining ParseFct.

functor ParseFct(
  structure SymTbl: SymTblSig
  structure Lex: LexSig
  sharing SymTbl.Sym = Lex.Sym
   and type SymTbl.Val.value = string) =
struct
  let val next = Lex.getsym()
  in SymTbl.update(table, next, “declared”) end
end

However, the let expression in the body is not type correct! Since the type of getsym() is Lex.Sym.sym, next has type Lex.Sym.sym. However, by the specification of update, its second argument must be of type SymTbl.Sym.sym. The problem is that although we have specified that SymTbl depends on a Sym structure and Lex depends on a Sym structure, nowhere have we specified that they depend on the same Sym structure. The type checker will not make an attempt to identify these two types, for the idea is that the functor should be applicable to any arguments that satisfy the formal parameter specification (not just those that satisfy the specification and in addition have extra sharing). Therefore one is allowed to specify needed sharing as well as needed components by a so-called SHARING SPECIFICATION. Grammatically, a sharing specification can occur anywhere amongst structure, type, value and exception specifications.

functor ParseFct(
  structure SymTbl: SymTblSig
  structure Lex: LexSig
  sharing SymTbl.Sym = Lex.Sym
   and type SymTbl.Val.value = string) =
struct
  let val next = Lex.getsym()
  in SymTbl.update(table, next, “declared”) end
end

One can specify sharing of structures and of types (but not of values or exceptions). In our example, we have to add yet a sharing specification, this time a type sharing specification.

2.7 Building the System

Notice that we have now written the code of the parser solely by declaring signatures and functors. We have not had to write a single top-level structure declaration. Having finished declaring the parser functor, we can return to the basics and declare functors that implement Sym and Val. These functors can be NULLARY, i.e. have an empty specification of formal parameters.

Exercise 13 Write nullary functors ValFct, SymFct and IntMapFct whose result match your signatures from the previous exercise.
We can now build the entire system by functor applications and top-level structure declarations.

structure Val = ValFct()
structure Sym = SymFct()
structure TTable =
  Symbucket(structure IntMap=IntMapFct()
    structure Val = Val
    structure Sym = Sym

structure Lex = LexFct(Sym)
structure Parser =
  ParseFct(structure SymTbl = TTable
    structure Lex = Lex)

The compiler will check that the sharing specified in the declaration of ParseFct really is met by the actual arguments.

Exercise 14 What is wrong with the following attempt to build the parser?

structure Val = ValFct()
structure TTable =
  Symbucket(structure IntMap=IntMapFct()
    structure Val = Val
    structure Sym = SymFct()

structure Lex = LexFct(SymFct())
structure Parser =
  ParseFct(structure SymTbl = TTable
    structure Lex = Lex)

2.8 Separate Compilation

Some ML implementations have facilities that allow you to compile declarations, for instance functor declarations, in such a way that the compiled code can persist between sessions. However, even without such a facility, using signatures and functors in the manner described above gives the valuable ability to separately compile modules consisting of signature and functor declarations, although the result of the compilation will not outlive the session.

Most ML systems have a use function which allows the ML source to be read from a file rather than from the terminal. One can then keep signatures in suitably named files and use these files at the beginning of each module.

use “symb.sig”;
use “val.sig”;
use “symtbl.sig”;
use “lex.sig”;
use “parse.sig”;

functor ParseFct(
  structure SymTbl: SymTblSig
  structure Lex: LexSig
  sharing SymTbl. Sym = Lex.Sym
  and type SymTbl. Val. value = string): ParseSig =
  struct
    ...
  end

In this way one avoids repeating the same signature declaration in many files (and thus also the problem of updating all copies if the signature is changed).
2.9 Good Style

It is good practice to keep signatures as small as possible. If one programs using functors and signatures as described above then writing the body of a functor will reveal which components of its formal parameters that particular functor needs to know about.

Different functors will need different details. Rather than gradually extending a single signature till it gets very large, one can use the include specification to enrich an existing signature.

signature SmallTbl = sig ... end

signature BigTbl =
    sig
        include SmallTbl
        datatype DebugInfo = ...
        val printInfo : unit->unit
    end

2.10 Bad Style

Signature declarations can contain free structure and type identifiers.

Structure declarations can contain free identifiers of any kind.

This allows you to write for example

structure Parser =
    struct
        structure Lex = Lex
        structure MyPervasives = MyPervasives
        structure ErrorReports = ErrorReports
        structure PrintFcns = PrintFcns
        structure Table = Table
        structure BigTable = BigTable
        structure Aux = Aux
    fun f(…) = … Table.lookup …
end

Here, the programmer has apparently made some effort to show that the parser depends on the structures listed at the beginning. However, if he has missed out a couple of structures from his list, it will have no effect on the declarations that follow, and so one does not as a reader feel confident that the list is exhaustive.

Moreover, when the reader wants to find out what the type of the lookup function is, he has to look in the declaration of the Table structure. In case Table is constrained by a signature, the search continues in the declaration of the signature. Otherwise, one will have to look at the code for lookup.

Most serious of all, when encountering the call of lookup one has no idea whether lookup has side effects that are important to other structures. In that case, the value of structuring code into structures and substructures is purely cosmetic. The only reliable help it gives you is a pointer to the structure in which the identifier is declared.

One particular horror is the misuse of open. Avaiable both in the core language and in the modules language, open S is a declaration which has the effect of adding all the bindings
of the structure $S$ to the current environment. This is helpful, if one has a single structure \textit{MyPervasives}, say, which is used everywhere in the project. But look at this:

\begin{verbatim}
structure Parser =
struct
 structure Lex = Lex
 open MyPervasives ErrorReports PrintFcns
 Table BigTable Aux

 fun f(...) = ... lookup ...
end
\end{verbatim}

Now finding \textit{lookup} is reduced to pure guesswork!

\textbf{Exercise 15} For each of the above points of criticism, consider to what extent it applies if one programs with signatures and functors only.
3 The Static Semantics of Modules

The purpose of this lecture is to explain the static semantics of modules. In particular, we shall look into the details of the crucial concepts signature matching and sharing.

3.1 Elaboration

Consider the two following signatures, the first of which stem from the MyTbl example of Lecture 1.

```ml
sig
type table
exception Lookup
val lookup: table * Identifier.sym -> Data.value
val update: table * Identifier.sym * Data.value -> table
end

sig
type table
exception Lookup
val lookup: table * string -> real
val update: table * string * real -> table
end
```

In one sense, these signatures are very different; the meaning of the first one depends on the free structures Data and Identifier, whereas the second depends on the pervasives only. However, if Identifier.sym happens to be string and Data.value happens to be real then the two expression are just different ways of expressing the same meaning. In that sense, the two signatures turn out to be equal.

To avoid such confusion concerning equality, it is often helpful to distinguish between a signature expression (the syntactic object) and a signature (its meaning). The transition from signature expressions to signatures is called elaboration. We use the word elaboration instead of evaluation, because, unlike evaluation, all elaboration can be done statically, by a compiler. The result of elaborating a signature expression depends on the meaning of the identifiers occurring free in the expression. In any given context, there are infinitely many signature expressions that elaborate to the same signature. It is even the case that in every context, every signature expression elaborates to infinitely many signatures, if it elaborates to any at all. However, among these there will always be some that are principal which means that, in a certain technical sense, all the others are instances of them, and one always takes a principal signature as the meaning of a signature declared at top-level.

Elaboration applies to structure expressions and functor declarations as well, yielding structures and functor signatures, respectively.

Essentially, the modules part of ML is a language for computing these (abstract) signatures, structures and functor signatures. The purpose of this lecture is to explain the principles that govern elaboration.

We shall not introduce a separate notation for structures, signatures and functor signatures. In many cases these are very similar to the expressions from which they were obtained, so we make do with the device of “decorating” expressions with so-called names. Names are semantic objects, completely distinct from identifiers; in the above examples, string and Identifier.sym are both identifiers which elaborate to the same type name.
3.2 Names

structure Stack =
struct
type elt = int
datatype stack = ST of elt list ref
val initStack = ST(ref[ ])
end

structure StackUser1 =
struct
structure Stack1 = Stack
...
end

structure StackUser2 =
struct
structure Stack2 = Stack
...
end

datastructure stack = ST of elt list ref

All the following sharing equations hold:
StackUser1.Stack1 = StackUser2.Stack2, elt
= int, Stack.stack = StackUser1.Stack1.stack.
None of the following sharing equations hold:
StackUser1 = StackUser2, Stack.stack
= StackUser2.stack

Sharing equations can be decided by decorating programs with NAMES. There are two kinds:

structure names: n1, n2, ...,
m1, m2, ...
type names: t1, t2, ...,
s1, s2, ...,
unit, int, bool, →

Two structures SHARE if they are decorated by the same structure name; two types
SHARE if they are decorated by the same type name.

3.3 Decorating Structures

Each elaboration of a structure expression of the form

struct ... end

yields a fresh structure, i.e. a structure decorated by a new name. Therefore such expressions are called GENERATIVE STRUCTURE EXPRESSIONS.

Each elaboration of a data type declaration (datatype ...) yields a fresh type i.e., a type decorated by a new name.

structure Stack_n1 =
struct
type elt_int = int
datastructure stack = ST of elt list ref
val initStack = ST(ref[ ])
end

structure StackUser1_n2 =
struct
structure Stack1_n1 = Stack
...
end

structure StackUser2_n38 =
struct
structure Stack2_n1 = Stack
...
datastructure stack = ST of elt list ref

To be complete, one would have to decorate each structure not merely by a name but also with its decorated components and subcomponents. (A structure expression in the form of a functor application does not in itself reveal the components of the resulting structure.) However, to keep decorations to a minimum, we shall usually not spell out the decorated subcomponents.
3.4 Decorating Signatures

signature StackSig(m₁, s₁, s₂) = 
sig
  type eltₘ₁
  type stackₕ₁
  val new_unit→ₕ₁ : unit→stack
end
signature TranspSig(m₁, s₁) = 
sig
  type eltₘ₁
  type stackₜ₁
  sharing type stackₜ₁ = Stack.stackₜ₁
  val new_unit→ₜ₁ : unit→stack
end

Bound names are collected at the signature identifier. They are listed between parenthesis to indicate that they are merely place holders.

The bound names of StackSig are m₁, s₁ and →. The free names of StackSig are unit and s₂. The bound names of TranspSig are m₁ and s₁. The free names of TranspSig are t₁, unit and →.

Exercise 16 Consider

signature Symbol =
sig
  type symbol
  type value
  sharing type value = int
end

Decorate this signature declaration with type and structure names. How many bound names are there? How many free?

If two structures are found to share by the static analysis then they really are the same at run-time. Therefore, when decorating signatures one must make sure that if two structures are made to share (by being given the same name) then any type or structure which is visible in both structures must be made to share as well.

Exercise 17 Consider the following signatures most of which you have already seen in Lecture 1.

signature ValSig =
sig
  type value
end
signature SymSig =
sig
  eqtype sym
  val hash : sym→int
end
signature LexSig =
sig
  structure Sym : SymSig
  val getsym : unit→Sym.sym
end
signature SymTblSig =
sig
  structure Val : ValSig
  structure Sym : SymSig
  type table
  val lookup:
    table * Sym.sym→ Val.value
  ...
end
signature ParseSig =
sig
  structure Lex : LexSig
  structure Tbl : SymTblSig
  sharing Lex.Sym = Tbl.Sym
type abstsyn
val parse : unit->abstsyn
end

Decorate these signatures. When one signature refers to another (for instance LexSig refers to SymSig) you should put a full decoration on the structure identifier (Sym), i.e. a decoration which shows both a name and the subcomponents of the structure. Full decorations can be drawn as trees; in the example at hand you can decorate Sym by

\[
\begin{array}{c}
\text{sym} \\
\text{s1} \\
\text{hash} \\
\text{s1} \rightarrow \text{int}
\end{array}
\]

Make sure that you decorate shared substructures (for instance Sym in ParseSig) consistently so as to represent that sharing of two structures implies sharing of their substructures.

3.5 Signature Instantiation

structure Stack\_n1 =
  struct
  type elt\_int = int
  datatype stack\_t1 = ST of elt list ref
  fun new\_unit\_\rightarrow\_t1 () = ST(ref[ ])
  end

signature StackSig\_A\_(m1,s1,s2) =
  sig
  m1 type elt\_s1
  datatype stack\_s2 = ST of elt list ref
  val new\_unit\_\rightarrow\_s2 : unit\_\rightarrow\_stack
  end

Note that if we substitute \text{n1} for \text{m1}, \text{int} for \text{s1} and \text{t1} for \text{s2} in the decoration of StackSig\_A then we get the decoration of Stack. We say that Stack is an instance of StackSig. More generally, we say that a structure is an instance of a signature if the decoration of the former is obtained from the decoration of the latter by performing a substitution of names for the bound names of the signature (the free names of the signature must be left unchanged). The process of substituting names for bound names is called REALISATION.
structure Stack,1 =
struct
  type elt,1 = int
datatype stack,1 = ST of elt list ref
fun new,unit→t1 () = ST(ref[ ])
end

signature StackSigB(m,1,s,1) =
sig
  m,1 type elt,s,1
datatype stack,s,1 = ST of elt list ref
sharing type stack,s,1 = Stack . stack,1
val new,unit→t1 : unit→stack
end

Stack is an instance of StackSigB via the realisation \{m,1↦n,1, s,1↦int\}.

structure OddStr,1 =
struct
  type elt,1 = int
  val test,1 : bool = false
end

signature WrongSig(m,1,s,1) =
sig
  m,1 type elt,s,1
  val test,s,1 : elt
end

OddStr is not an instance of WrongSig, for s,1 would have to be realised by int (because of elt) but then test is decorated by int in the signature and by bool in the structure.

3.6 Signature Matching

Matching of a structure against a signature is a combination of two operations. The first, signature instantiation (described above), is concerned with instantiating the bound names of the signature to the “real” names of the structure. The second is concerned with ignoring information in the structure which is not required by the signature.

structure Tree,1 =
struct
  datatype 'a tree,1 = LEAF of 'a
  | NODE of 'a tree * 'a tree
type intTree,1 = int tree
fun max(a:int, b:int) =
  if a > b then a else b
fun depth,tree,1→int (LEAF _) = 1
| depth(NODE(left, right))=
  max(depth left, depth right)
end

signature TreeSig(m,1,s,1,s,2) =
sig
  m,1 type 'a tree,s,1
  type intTree,s,2
  fun depth,s,2→int : intTree→int
end

A structure MATCHES a signature if the structure can be cut down to an instance of the signature by

1. forgetting components;
2. forgetting polymorphism of variables.

Tree matches TreeSig. First perform the realisation \{m,1↦n,1, s,1↦t,1, s,2↦int t,1\} on the signature. The resulting decoration can be obtained from the decoration of Tree by

1. forgetting the constructors LEAF and NODE
2. instantiating 'a t,1→int to int t,1→int (i.e. the realisation of s,2→int)
Exercise 18  Let mytype be a type which is declared in a structure and specified in a signature. In which of the following cases can the structure match the signature?

1. mytype is declared as a datatype and specified as a datatype.
2. mytype is declared as a datatype and specified as a type;
3. mytype is declared as a type and specified as a type;
4. mytype is declared as a type and specified as a datatype.

3.7 Signature Constraints

structure Tree: TreeSig =
  struct ... end

In a structure declaration with an explicit signature constraint, the resulting view of the declared structure is precisely the one given by the instantiated signature.

In the example above, the resulting view of Tree will hide the constructors NODE and LEAF and the function max. Notice, however, that intTree is decorated by the instance of s2, i.e. by int t1, where t1 is the decoration of 'a tree. Consequently, Tree.intTree and int Tree.tree now mean the same thing, namely int t1. This sharing was obtained through realisation — it was not explicit in TreeSig.

In short, explicit signature constraints can remove components and polymorphism, but they do not affect existing sharing.

Here is an example of a structure declared with an explicit signature constraint. You should convince yourself that the structure really does match the signature.

signature SymSig =
sig
type sym
  type code
  sharing type code = int
  val hash : sym->int
  val mksym : string->sym
  val nameof : sym->string
end

structure Sym : SymSig =
  struct
datatype sym = SYM of string * int
type code = int
  fun convert(s: string): code = ...
  fun hash(SYM(s, n))= n
  fun mksym(s) = SYM(s, convert s)
  fun nameof(SYM(s, _))= s
end

Exercise 19  Complete the declaration of convert.

Exercise 20  Declare a different structure NewSym, also constrained by SymSig, such that NewSym.sym shares with string. Which of the following expressions are valid?

1. "a" ^ NewSym.mksym "d"
2. "a" ^ Sym.mksym "d"
3.8 Decorating Functors

Dynamically, the body of a functor is not evaluated when the functor is declared but it is evaluated once for each time the functor is applied.

\[
\text{functor } \text{StackFct}() = \\
\text{struct} \\
\text{datatype stack} = \text{ST of int list ref} \\
\text{val data} = \text{ST(ref [ ])}
\]

\[
\text{function StackFct}() = \\
\text{struct} \\
\text{datatype stack} = \text{ST of int list ref} \\
\text{val data} = \text{ST(ref [ ])}
\]

\[
\text{structure Stack1} = \text{StackFct}() \\
\text{structure Stack2} = \text{StackFct}()
\]

Since the two applications of \text{StackFct} create two distinct references, \text{Stack1} and \text{Stack2} are different and must not be seen to share.

Now let us consider the problem of decorating \text{StackFct} with names. We start out by decorating the body in the usual way. However, each time we need a fresh name, we record it at the \text{=} in the first line. The names that hence are accumulated are called the GENERATIVE NAMES of the functor. The generative names are bound in the sense that they stand as place holders for fresh names which we choose when we eventually apply the functor. (Like the bound names in signatures, we write generative names between parenthesis; unlike the bound names of signatures, generative names are written on the right — because they concern the right side only.)

In the case of nullary functors, i.e. functors that take an empty argument, the structure resulting from a functor application is decorated by taking the decoration of the functor body with each generative name replaced by a fresh name.

\[
\text{functor StackFct}() =_{(m_1,s_1)} \\
\text{struct}_{m_1} \\
\text{datatype stack}_{s_1} = \text{ST of int list ref} \\
\text{val data}_{s_1} = \text{ST(ref [ ])}
\]

Exercise 21 The decorations of \text{Stack1} and \text{Stack2} show the top-most structure name only. Complete the decorations.

Notice that \text{Stack1} and \text{Stack2} do not share; not even the types \text{Stack1.stack} and \text{Stack2.stack} share. Consequently, the variables \text{Stack1.data} and \text{Stack2.data} have different types and so the type checker prevents one from mistaking the one for the other.

3.9 External Sharing

Within the body of a functor one may refer to identifiers (of any kind) declared in the context of the functor. Such identifiers are said to occur FREE in the functor. This results in EXTERNAL SHARING, i.e. a decoration in which some of the names stem from outside the functor.
structure MyPervasives =
struct
    datatype num_t1 = NUM of int
end

functor StackFct′() = (m2)
struct
    structure MyPer_n1 = MyPervasives
    type stack_t1 list ref = MyPer_n1.num list ref
    val data_t1 list ref : stack = ref [ ]
end

structure Stack1′ n8 = StackFct′()
structure Stack2′ n9 = StackFct′()

Notice that external names are not generative; they are left unchanged when the functor is applied.

Exercise 22 Which of the following sharing equations hold?
Stack1′ = Stack2′;
Stack1′.MyPer = Stack2′.MyPer;
type Stack1′.stack = Stack2′.stack.

3.10 Functors with Arguments

signature SymSig(m1,s1) =
sig
    eqtype sym_s1
end

functor SymDir(Sym: SymSig) = (m2,s2)
struct
    datatype dir_s2 = DIR of Sym.sym->int
    fun update . . .
end

When decorating the body of a functor which has an argument, we assume that we have a structure (by the name of the formal parameter) which precisely matches the parameter signature. We assume neither more components nor more sharing than is specified in the signature, for we want the functor to be applicable to all actual argument structures that match the formal parameter signature.

In the above example we simply assume that the name of Sym is m1 and that the name of Sym.sym is s1 (as those names are not used of free structures elsewhere; in general, one might have to rename some of the bound names. Having used m1 and s1 we simply start generating names from m2 and s2 in the body.

structure Actual_n1 =
struct
    type sym_string = string
end

structure Result_n2 = SymDir(Actual)
Result receives a fresh structure name and Result.dir a fresh type name. Note that Actual matches SymSig.

3.11 Sharing Between Argument and Result

signature SymSig\(_{(m_1, s_1)}\) =
  sig\(_{m_1}\)
  eqtype sym\(_{s_1}\)
end

functor SymDir(Sym: SymSig) \(=(m_2)\)
  struct\(_{m_2}\)
    type dir\(_{s_1\rightarrow int}\) = Sym.sym\(\to\)int
    fun update ...
  end

The type name \(s_1\) is shared between the argument and the body. When the functor is applied, this sharing must be translated into sharing between the actual argument and the actual result.

structure Actual\(_{n_1}\) =
  struct type sym\(_{string}\) = string
end

structure Result\(_{n_2}\) = SymDir(Actual)

Exercise 23 Complete the decoration of Result.

Exercise 24 (1) Using the latest definition of SymDir, is the following expression legal?

\[
\text{fn (d: Result.dir) => d "]abc"}
\]

(2) Same question, but for the earlier definition of SymDir.

A full decoration of the result of applying a functor with one argument can be obtained as follows:

1. Match the actual argument against the formal parameter signature yielding a realisation which maps bound names to formal signature to names in the actual argument;

2. Apply this realisation to the decoration of the functor body;

3. also substitute fresh names for the generative names of the functor body.

3.12 Explicit Result Signatures

When a functor declaration contains a result signature, the decoration of the functor declaration proceeds as follows:

1. decorate the functor without the result signature;

2. decorate the result signature. If one can get an instance of the result signature by removing components and polymorphism from the decorated body, then this instance is used as a formal result instead of the decorated body; otherwise the declaration is rejected.

This has the effect that upon application of the functor, sharing is propagated as before, but only the components and polymorphism of the result signature are visible in the actual result.

Exercise 25 Why is it not always the case that obtaining \(R\) by
functor \( F(S: SIG): SIG' = \)
struct...end

structure \( R = F(\ldots) \)

is equivalent to obtaining \( R \) by

functor \( F(S: SIG) = \)
struct...end

structure \( R: SIG' = F(\ldots) \)

Give a condition on \( SIG' \) under which this difference disappears.